

Thermodynamics of system with density- and/or temperature-dependent mass particles

Shaoyu Yin¹ and Ru-Keng Su^{1,2*}

1. *Department of Physics, Fudan University,
Shanghai 200433, China*

2. *CCAST(World Laboratory),
P.O.Box 8730, Beijing 100080, China*

Abstract

The thermodynamics with medium effects expressed by the dispersion relation of the temperature and density dependent particle mass is studied. Many previous treatments have been reviewed. A new thermodynamical treatment based on the equilibrium state is suggested. Employing the quark mass density- and temperature-dependent model, the discrepancies between our treatment and others are addressed.

PACS numbers: 24.85.+p;12.39.-x;05.70.Ce

* rkusu@fudan.ac.cn

I. INTRODUCTION

It is generally accepted that the effective masses of particles will change with temperature and density due to medium effects. Many theoretical considerations, including the finite temperature QHD model[1], Brown-Rho scaling[2], QCD sum rules[3], finite temperature QMC model[4, 5], vacuum polarization Feynman diagrams calculations[6, 7, 8] and *etc.*, had been suggested to investigate the effective masses of mesons and nucleons. By means of the Thermo-field Dynamics, after summing the temperature- and density-dependent vacuum polarization diagrams for π , σ , ω , ρ mesons and the three-line vertex diagrams which give the corrections of the $NN\pi$, $NN\sigma$, $NN\omega$, $NN\rho$ couplings respectively, we have extended the one-sigma[9], one-pion[10], one-omega[11] and one-rho exchange potentials[6, 8, 12], and then the Bonn potentials[13] to finite temperature and density. We have found that the attractive part of the NN interaction becomes weaker and the repulsive part becomes stronger when the temperature and/or density of the system increase. This is of course very reasonable.

Besides theoretical study, many experimental results which predict the changes of particle masses with temperature and density have been shown. In particular, the experiments of TAGX collaboration have shown directly that when the density of the nucleon medium equals to $0.7n_0$ where n_0 is the saturation density, the effective mass of neutral ρ -meson reduces to 610MeV[14]. Both theoretical and experimental results confirm that the medium effects are important for studying the nuclear or quark systems.

To illustrate the medium effects more transparently in the theory instead of the first principle calculation, many authors introduced different hypothesis to represent the medium contributions to particle masses, for example, introduced the density-dependent vacuum energy $B(n)$ to modify QMC model[15, 16], suggested the density-dependent $NN\rho$ coupling to address liquid-gas phase transition[17], supposed the u , d and s quark masses depends on density to mimic the quark confinement mechanism[18], introduced temperature-dependent vacuum energy $B(T)$ to modify the quark mass density-dependent (QMDD) model[19], and *etc.* Employing these hypothesis, many physical properties of nuclear matter, quark matter, nucleon system and hyperon system had been discussed.

Although the density- and temperature-dependent particle masses $m^*(T, n)$ can mimic the medium effects, when we discuss the thermodynamical behaviors of the system with such particles, many difficulties will emerge. First, the dispersion relation for a particle with

energy ϵ and momentum k becomes

$$\epsilon(k, T, n) = [k^2 + m^*(T, n)]^{1/2} \quad (1)$$

due to the medium effect. It was shown in Ref.[20] that the usual fundamental thermodynamical partial derivative relation is not satisfied for such system, we must add the additional terms. Secondly, it is well known in thermodynamics that if we choose a pair of independent variables, for example, temperature T and volume V , and give the suitable characteristic thermodynamical function, for example, Helmholtz free energy $F = F(T, V)$, then all thermodynamical quantities can be obtained by the partial derivatives of $F(T, V)$, because

$$dF = -SdT - pdV, \quad (2)$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V, \quad p = -\left(\frac{\partial F}{\partial V}\right)_T. \quad (3)$$

Eq.(3) give the entropy and the equation of state respectively. Other thermodynamical quantities such as internal energy U , enthalpy H , Gibbs function G , heat capacities c_V and c_p can be obtained from Eq.(2) by derivatives. In above calculations, no integral process will be needed and no integral constant has to be determined. In fact, this is the reason why all calculation of thermodynamical quantities in the canonical ensembles can be done with the calculation of the partition function $Z = Z(T, V)$, because the partition function Z is related to free energy F directly. For systems with three independent variables, say, T , V and chemical potential μ , the suitable characteristic function is the thermodynamical potential $\Omega = \Omega(T, V, \mu)$ because

$$d\Omega = -SdT - pdV - Nd\mu. \quad (4)$$

But for system with density-dependent mass particle, Ω is not only a function of T , V , μ , but also of the density n , because $\Omega = \Omega(T, V, \mu, m^*(T, n))$. How to treat the thermodynamics self-consistently is still a serious problem and has many wrangles in present references. We will give a brief review of various treatments and show their contradictory in the next section. A few comments on different treatments will also be presented there.

This paper evolves from an attempt to suggest a new thermodynamical treatment to study the system with density- and temperature-dependent mass particles. Since the particle mass depends on temperature and density, a lot of ambiguities will happen when one uses the partial derivatives along a reversible process to obtain the thermodynamical quantities.

To avoid this difficulty, instead of a *reversible process*, we argue that we can calculate the thermodynamical quantities at an *equilibrium state*. In fact, all the physical quantities, such as p , U , F , H , G , are the functions of equilibrium state and have definite values respectively. We can use the relations between thermodynamical functions to obtain above quantities provided that the thermodynamic potential Ω can be obtained. We will employ the QMDD model to explain our treatment in detail in Sec.III. By means of quark mass density- and temperature-dependent (QMDTD) model, we will show our results and the comparison to other treatments in Sec.IV. The last section is a summary.

II. DIFFERENT THERMODYNAMICAL TREATMENTS

There have been several different kinds of treatments dealing with the thermodynamics for the system with temperature- and/or density-dependent mass particles. The basic difficulty comes from Eq.(1) while one calculate the partial derivative of thermodynamical functions along a reversible process. Though several authors had still employed the usual direct formulae to calculate the thermodynamical quantities and shown[21, 22]

$$P = -\frac{\Omega}{V}, \quad (5)$$

$$n_i = -\frac{1}{V} \left(\frac{\partial \Omega}{\partial \mu_i} \right)_{T, n_B}, \quad (6)$$

$$\varepsilon = \frac{\Omega}{V} + \sum_i \mu_i n_i - \frac{T}{V} \left(\frac{\partial \Omega}{\partial T} \right)_{\{\mu_i\}, n_B}, \quad (7)$$

where n_B is the baryon density, n_i is the number density of particle i , μ_i is the corresponding chemical potential, and ε is the energy density. Their failure is obviously because, with the temperature- and density-dependent mass term, Ω is no longer an explicit function of T , V , μ_i . The usual direct partial derivative formulae are not applicable. To overcome this difficulty, many different treatments or methods have been suggested in the market, which can be generally categorized into two kinds.

(I) The first kind of treatments takes the mass term as a function of density and/or temperature and make a great effort to work through the derivatives of a composite function along a reversible process. In deriving the energy density and the pressure, extra terms emerge in this approach due to the dependence of the mass on the density and/or temperature. Although many authors work along this direction to study the mass density and/or temperature dependence, their formulae and results are very different with each other.

(IA). For the the density-dependent mass, in Ref.[23], they gave

$$P = -\frac{1}{V} \left(\frac{\partial(\Omega/n_B)}{\partial(1/n_B)} \right)_{T, \{\mu_i\}} = -\frac{\Omega}{V} + \frac{n_B}{V} \left(\frac{\partial\Omega}{\partial n_B} \right)_{T, \{\mu_i\}}, \quad (8)$$

$$\varepsilon = \frac{\Omega}{V} - \frac{n_B}{V} \left(\frac{\partial\Omega}{\partial n_B} \right)_{T, \{\mu_i\}} + \sum_i \mu_i n_i - \frac{T}{V} \left(\frac{\partial\Omega}{\partial T} \right)_{\{\mu_i\}, n_B}, \quad (9)$$

where n_i still satisfies Eq.(6). The extra terms in Eqs.(8, 9) produce significant changes in the energy density ε , and makes that the pressure become negative in low density regions.

(IB). In Ref.[24, 25], for the density-dependent mass, the pressure and the energy density become

$$P = -\frac{\Omega}{V} + \frac{n_B}{V} \left(\frac{\partial\Omega}{\partial n_B} \right)_{T, \{\mu_i\}}, \quad (10)$$

$$\varepsilon = \frac{\Omega}{V} + \sum_i \mu_i n_i - \frac{T}{V} \left(\frac{\partial\Omega}{\partial T} \right)_{\{\mu_i\}, n_B}. \quad (11)$$

They use the same pressure formula as that of (IA) Eq.(8), but do not agree with the expression in Eq.(9), because it cannot give a correct QCD vacuum energy. Then they use the same energy density formula as Eq.(7).

(IC). In Ref.[26], for the density- and temperature-dependent mass, the pressure and the energy density read

$$P = -\tilde{\Omega} - V \frac{\partial\tilde{\Omega}}{\partial V} + n_B \sum_i \frac{\partial\tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial n_B}, \quad (12)$$

$$\varepsilon = \tilde{\Omega} - \sum_i \mu_i \frac{\partial\tilde{\Omega}}{\partial \mu_i} - T \frac{\partial\tilde{\Omega}}{\partial T} - T \sum_i \frac{\partial\tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T}, \quad (13)$$

where $\tilde{\Omega} = \Omega/V$ stands for the density of the thermodynamical potential.

(II) The second kind of treatments add different terms to the system to keep the usual thermodynamical derivative relations unchanged. For example, in order to make the Eq.(4) and the calculation of the thermodynamical potential self-consistently, they add extra terms to the Hamiltonian or to the thermodynamical potential. These additional terms differ for different authors.

(IIA). To keep the fundamental thermodynamic relations

$$\varepsilon(T) = T \frac{dp(T)}{dT} - p(T), \quad s = \left(\frac{\partial p}{\partial T} \right)_\mu, \quad n = \left(\frac{\partial p}{\partial \mu} \right)_T \quad (14)$$

self-consistently, in Ref.[20], a term E_0^* had been added to the Hamiltonian. This term is determined by the condition

$$\left(\frac{\partial p}{\partial c_i} \right)_{T, \mu, \{c_{j \neq i}\}} = 0, \quad (15)$$

where $\{c_i\}$ is the temperature- and density-dependent terms in the Hamiltonian. Then the pressure and energy density become

$$p(T, \mu, \{c_i\}) = \mp \frac{gkT}{2\pi^2} \int k^2 dk \ln[1 \mp \exp^{-\beta(\epsilon(k)-\mu)}] - B^*, \quad (16)$$

$$\varepsilon(T, \mu, \{c_i\}) = \frac{g}{2\pi^2} \int \frac{\epsilon(k)k^2 dk}{\exp^{-\beta(\epsilon(k)-\mu)} \mp 1} - B^*, \quad (17)$$

where

$$B^* = \lim_{V \rightarrow \infty} \frac{E_0^*}{V}, \quad (18)$$

and g is the degeneracy factor.

(IIB). In Ref.[27], an extra term $\Omega_a(n_B)$ had been added to the thermodynamical potential,

$$\tilde{\Omega} = - \sum_i \frac{g_i T}{(2\pi)^3} \int dk^3 \ln(1 + e^{-\beta(\epsilon_i(k)-\mu_i)}) + \Omega_\alpha(n_B), \quad (19)$$

where $\Omega_a(n_B)$ is determined by the constraint

$$\left. \frac{\partial \tilde{\Omega}}{\partial n_B} \right|_{\{\mu_i\}} = 0. \quad (20)$$

At zero temperature, the corresponding thermodynamical formulae become

$$p = -\tilde{\Omega}, \quad (21)$$

$$\begin{aligned} \varepsilon = & \sum_i \frac{g_i}{48\pi^2} \{ \mu_i [\mu_i^2 - m_i^2(n_B)]^{1/2} [6\mu_i^2 - 3m_i^2(n_B)] \\ & - 3m_i^4(n_B) \ln \left[\frac{\mu_i + [\mu_i^2 - m_i^2(n_B)]^{1/2}}{m_i(n_B)} \right] \} + \Omega_\alpha(n_B), \end{aligned} \quad (22)$$

$$n_i = \frac{g_i}{6\pi^2} [\mu_i^2 - m_i^2(n_B)]^{3/2}. \quad (23)$$

In summary, from above treatments we come to a conclusion that how to treat the thermodynamics with the medium effect is still a serious problem and has made many wrangles in present references[21, 22, 23, 24, 25, 26]. In fact, many treatments are contradict with each other. To show their confusion and ambiguity, we give a few comments in the following:

(1). The first argument is that the dispersion relation Eq.(1) and the derivative formula of thermodynamical potential Eq.(4) correspond to different conditions. In Eq.(1), the medium effect has been taken into account. This effect makes that the particle mass

becomes a temperature- and density-dependent effective mass. But Eq.(4) corresponds to the thermodynamical system only where Ω, S, T, p are the thermodynamical quantities of the system respectively. They do not include the medium effect. In order to make Eq.(1) and the derivative formula of thermodynamical process self-consistent, instead of Eq.(4), we must establish a formula of thermodynamical process for the total system. It includes the variations of the quantities not only for the system, but also for the medium. In this formula, all extensive quantities must be replaced by the quantity of the total system, for example, $S \rightarrow S_s + S_m$, where S_s and S_m are the entropy of the system and the medium respectively. The problem is that so far we do not know how to calculate S_m and other extensive thermodynamical quantities of the medium directly.

(2). To illustrate above argument more transparently, we imagine two systems. These two systems are almost identical: their only difference is that one system consists of constant mass (m_1) particles while the other of temperature- and density-dependent mass ($m_2(T, n)$) particles. Fixed the temperature and density as T_0 and n_0 respectively, which satisfy

$$m_2(T_0, n_0) = m_1, \quad (24)$$

at the equilibrium state with (T_0, n_0) , obviously, the two system are completely the same. They arrive at and stay in the same equilibrium state and have the same thermodynamical quantities, such as the pressure and the energy density, which are just functions of equilibrium state. But if we use equations with partial derivatives to calculate these quantities of the later system, extra terms, such as $\frac{\partial \tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial n_B}$ and $\frac{\partial \tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T}$ in Eqs.(12, 13), emerge. These terms will never appear in the calculation of the former system since in this system m_1 is a constant. This simple example tells us that the first kind of treatments is not right because the medium effect has been neglected in Eq.(4).

(3). Now we hope to give a brief comment on the second kind of treatments. They hope to add an extra term to the system to consider the medium effect. But unfortunately, in general, the extra term cannot be determined by an additional constraint in terms of partial derivatives uniquely. It cannot be expressed as a zero value of thermodynamical functions, because the effective mass $m^*(T, n)$ comes from the interaction of the particle and the medium. This interaction depends on temperature and density. As an example, let's discuss the treatment (IIB). Obviously, if we add an arbitrary temperature function $f(T)$ to $\Omega_\alpha(n_B)$, the new expression for total $\tilde{\Omega}$ still satisfies the constraint Eq.(20), but the

additional arbitrary function of T will change the thermodynamical functions, such as the entropy, which depend on the derivation of temperature.

III. NEW TREATMENT BASED ON THE EQUILIBRIUM STATE

The above arguments impress us to give up the derivative calculation of thermodynamical quantities along a reversible process, since Eq.(4) dose not include the medium effect. Instead of studying the reversible process, we focus our attention on equilibrium states. We suggest a new treatment which is based on the equilibrium state in this section. We will show there is no ambiguity in our treatment.

According to the thermodynamics of equilibrium state, all thermodynamical functions such as Ω , S , U , G , p ... have definite values respectively at an equilibrium state. If we can calculate the thermodynamical potential Ω and the Gibbs' function G for a fixed equilibrium state, we can use the following definitions to find other thermodynamical quantities:

$$U = \sum_i n_i \epsilon_i, \quad (25)$$

$$n = \frac{N}{V} = \sum_i n_i = \frac{g_i}{e^{\beta(\epsilon_i - \mu)} \pm 1}, \quad (+ \text{ for Fermion and } - \text{ for Boson}) \quad (26)$$

$$G = \sum_i n_i \mu_i \quad (27)$$

$$F = \Omega + G, \quad (28)$$

$$S = \frac{U - F}{T}, \quad (29)$$

$$P = -\frac{\Omega}{V}. \quad (30)$$

To show our treatment explicitly, we employ the QMDTD model [19, 28, 29, 30, 31]. The QMDTD model is extended from the QMDD model which was first suggested by Fowler *et al.* many years ago [18]. According to the QMDD model, the masses of u , d quarks and s quark are given by

$$m_q = \frac{B}{3n_B}, \quad (q = u, \bar{u}, d, \bar{d}), \quad (31)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B}{3n_B}, \quad (32)$$

where B is the vacuum energy density and m_{s0} is the current mass of the strange quark. It is clear that the masses of quarks become infinity when the baryon density goes to zero, which

means that the quark confinement in this model is permanent. To remove the permanent confinement of quark, we modified the QMDTD model by introducing [19, 28]

$$m_q = \frac{B(T)}{3n_B}, \quad (q = u, \bar{u}, d, \bar{d}), \quad (33)$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B(T)}{3n_B}, \quad (34)$$

where

$$B(T) = B_0[1 - (\frac{T}{T_c})^2]. \quad (35)$$

The quark mass depends on both density and temperature, so it is called a QMDTD model. We have used this model to study the properties of strange quark matter (SQM) [28, 29, 30, 31].

For the system of SQM, in which the masses of quarks satisfy Eqs.(33-35), at equilibrium state, the thermodynamical potential of the system reads [23, 26, 28]

$$\Omega = - \sum_i \frac{g_i TV}{(2\pi)^3} \int d^3k \ln(e^{-\beta(\epsilon_i(k) - \mu_i)} + 1), \quad (36)$$

The internal energy U and the Gibbs' function G are

$$U = \sum_i g_i \int d^3k \frac{\epsilon_i(k)}{e^{-\beta(\epsilon_i(k) - \mu_i)} + 1}, \quad (37)$$

$$G = \sum_i g_i \int d^3k \frac{\mu_i}{e^{-\beta(\epsilon_i(k) - \mu_i)} + 1}, \quad (38)$$

respectively. Other thermodynamical quantities can be obtained by Eqs.(25-30).

At finite temperature, the antiquarks must be considered. The baryon density satisfies

$$n_B = \frac{1}{3}(\Delta n_u + \Delta n_d + \Delta n_s), \quad (39)$$

where

$$\Delta n_i = n_i - n_{\bar{i}} = \frac{g_i}{(2\pi)^3} \int d^3k (\frac{1}{\exp[\beta(\epsilon_i(k) - \mu_i)] + 1} - \frac{1}{\exp[\beta(\epsilon_i(k) + \mu_i)] + 1}), \quad (40)$$

($n_{\bar{i}}$) n_i is the number density of the (anti)flavor i ($i = u, s, d$), $g_i = 6$, for antiquark $\mu_{\bar{i}} = -\mu_i$. Following Ref. [22, 28], the system of SQM must satisfy the constraints

$$\mu_s = \mu_d = \mu_u + \mu_e, \quad (41)$$

because inside SQM, s (and \bar{s}) quarks are produced through the weak process

$$u + d \leftrightarrow u + s, \quad s \rightarrow u + e^- + \bar{\nu}_e, \quad d \rightarrow u + e^- + \bar{\nu}_e, \quad u + e^- \rightarrow d + \nu_e, \quad (42)$$

and similarly for antiquarks. The condition of charge neutrality reads

$$2\Delta n_u = \Delta n_d + \Delta n_s + 3\Delta n_e. \quad (43)$$

IV. RESULTS AND DISCUSSION

For the convenience of comparison, our numerical calculations have been done by adopting the parameters $B_0 = 170\text{MeVfm}^{-3}$, $m_{s0} = 150\text{MeV}$ and $T_c = 170\text{MeV}$, as that of Ref.[28]. Our results are summarized in Figs.1-4 and Table I.

The energy per baryon vs. baryon number density n_B is shown in Fig.1 where the temperature is fixed at 50MeV . In Fig.1, the solid line refers to the present treatment and other four dashed lines refer to treatments in Ref.[21, 22], IA, IB and IC respectively. We see the solid line is lower than the others. The saturation points for different treatments are summarized in Table I. We find from Fig.1 and Table I that the differences are remarkable for different treatments.

To illustrate the thermodynamical characters of our treatment, we show the internal energy and the free energy curves calculated by our treatment in Fig.2 respectively. The energy per baryon ε/n_B (solid lines) and the free energy per baryon f/n_B (dashed lines) vs. baryon density for different temperatures $T = 0, 30, 50, 100\text{ MeV}$ are shown in Fig.2. We find that ε/n_B increases and f/n_B decreases as the temperature increases. The saturation points exist for all temperatures.

The equation of state for different treatments are shown in Fig.3, where the solid line refers to present treatment and other lines for previous treatments, respectively, as labeled in the figure. The solid line exhibits a significant property which differs from that of the lines of treatments IA, IB and IC. The pressure is definitely positive in our treatment. But for IA, IB and IC treatments, the pressure becomes negative in the small energy density regions. The pressure will not be negative due to its thermodynamical treatment for a system with positive energy. This result confirms that our treatment is correct.

treatments	$n_{B0}(fm^{-3})$	ε/n_B
in refs.[21,22]	0.46	1023
IA	0.55	1083
IB	0.36	1120
IC	0.34	1084
our new treatment	0.45	1007

TABLE I: Saturation points at $T = 50\text{MeV}$ for different treatments.

In Ref.[26], the authors claimed that the dispersion relation of effective mass m^* must satisfy the constraint

$$\lim_{T \rightarrow 0} \frac{\partial m^*}{\partial T} = 0, \quad (44)$$

based on their formula for entropy

$$s = -\frac{\partial \Omega}{\partial T} - \sum_i \frac{\partial \Omega}{\partial m_i} \frac{\partial m_i}{\partial T}. \quad (45)$$

If $\lim_{T \rightarrow 0} \frac{\partial m^*}{\partial T} \neq 0$, it will conflict with the third law of thermodynamics, $\lim_{T \rightarrow 0} S = 0$. We hope to point out that this argument is not right because it is based on Eq.(4) which has not taken the medium effect into account. In Fig.4 we draw the entropy per baryon vs. temperature curves for different baryon densities $n_B = 0.2, 0.5, 1.0 \text{ fm}^{-3}$ with dispersion relations $m_1^*(T, n_B) = \frac{B_0(1-T/T_c)}{3n_B}$, using our treatment. We even choose three different dispersion relations $m_1^*(T, n_B) = \frac{B_0(1-T/T_c)}{3n_B}$, $m_2^*(T, n_B) = \frac{B_0(1-T^2/T_c^2)}{3n_B}$ and $m_3^*(T, n_B) = \frac{B_0}{3n_B}$ for the same baryon density $n_B = 0.5 \text{ fm}^{-3}$ and find that all curves get together at the point $T = 0$, $S = 0$. It means that our treatment is consistent with the third law of thermodynamics no matter how the dispersion relation is. The constraint Eq.(44) needs not to be satisfied.

V. SUMMARY

In summary, we have shown the shortcomings of the previous treatments, which based on the partial derivatives of thermodynamical functions along a reversible process or based on the additional terms to thermodynamical potential. The previous treatments obstruct the correct consideration of medium in thermodynamics. A new treatment of medium thermodynamics based on equilibrium state is suggested. Employing QMDTD model, we address the discrepancies between our treatment and others in Fig.1-4 and Table I. We find that the negative pressure and the constraint condition Eq.(44) are removed in our treatment.

Acknowledgements

This work is supported in part by NNSF of China and the National Basic Research Programme of China.

- [1] B. D. Serot and J. D. Walecka, Inter. Jour. Mod. Phys. **E6**, 515 (1997) and refs. herein.
- [2] G. E. Brown and M. Rho, Phys. Rev. Lett **66**, 2720 (1993).
- [3] T. Hatsuda, Y. Koiko and S. H. Lee, Nucl. Phys. **B394**, 221 (1993).
- [4] H. Q. Song and R. K. Su, Phys. Lett. **B358**, 179 (1995); J. Phys. **G22**, 1025 (1996).
- [5] K. Saito, K. Tsushima and A. W. Thomas, Phys. Rev. **C56**, 566 (1997).
- [6] Y. J. Zhang, S. Gao, and R. K. Su, Phys. Rev. **C56**, 3336 (1997).
- [7] H. Shiomi and T. Hatsuda, Phys. Lett. **B334**, 281 (1994).
- [8] P. Wang, Z. W. Chong, R. K. Su and P. K. N. Yu, Phys. Rev. **C59**, 928 (1999).
- [9] R. K. Su and G. T. Zheng, J. Phys. **G16**, 203 (1990).
- [10] Z. X. Qian, C. G. Su and R. K. Su, Phys. Rev. **C47**, 877 (1993); R. K. Su, G. T. Zheng and G. G. Siu, J. Phys. **bf G19**, 79 (1993).
- [11] R. K. Su, S. J. Yang, S. Gao and P. K. N. Yu, J. Phys. **G20**, 1757 (1994).
- [12] P. Wang and R. K. Su, Chinese Phys. Lett. **16**, 793 (1999).
- [13] S. Gao, Y. J. Zhang and R. K. Su, Phys. Rev. **C53**, 1098 (1996).
- [14] G. J. Lolos *et. al.*, Phys. Rev. Lett. **80**, 241 (1998).
- [15] X. Jin and B. K. Jennings, Phys. Rev. Lett. **B374**, 13 (1996); Phys. Rev. **C54**, 1427 (1996).
- [16] P. Wang, R. K. Su, H. Q. Song and L. L. Zhang, Nucl. Phy. **A653**, 166 (1999).
- [17] W. L. Qian, R. K. Su and P. Wang, Phys. Lett. **B491**, 90 (2000); W. L. Qian, R. K. Su and H. Q. Song, Phys. Lett. **B520**, 217 (2001).
- [18] G. N. Fowler, S. Raha and R. M. Weiner, Z. Phys. **C9**, 271 (1981).
- [19] Y. Zhang, R. K. Su, S. Q. Ying and P. Wang, Europhys. Lett. **56**, 361 (2001).
- [20] M. I. Gorenstein and S. N. Yang, Phys. Rev. D **52**, 5206 (1995).
- [21] S. Chakrabarty, S. Raha, and B. Sinha, Phys. Lett. B **229**, 112 (1989).
- [22] S. Chakrabarty, Phys. Rev. D **43**, 627 (1991); *ibid.* **48**, 1409 (1993).
- [23] O. G. Benvenuto and G. Lugones, Phys. Rev. D **51**, 1989 (1995); G. Lugones and O. G.

- Benvenuto, *ibid.* **52**, 1276 (1995).
- [24] G. X. Peng, H. C. Chiang, P. Z. Ning, B. S. Zou, Phys. Rev. C **59**, 3452 (1999).
 - [25] G. X. Peng, H. C. Chiang, B. S. Zou, P. Z. Ning, and S. J. Luo, Phys. Rev. C **62**, 025801 (2000).
 - [26] X. J. Wen, X. H. Zhong, G. X. Peng, P. N. Shen, and P. Z. Ning, Phys. Rev. C **72**, 015204 (2005).
 - [27] P. Wang, Phys. Rev. C **62**, 015204 (2000).
 - [28] Y. Zhang and R. K. Su, Phys. Rev. C **65**, 035202 (2002).
 - [29] Y. Zhang and R. K. Su, Phys. Rev. C **67**, 015202 (2003).
 - [30] C. Wu, W. L. Qian and R. K. Su, Phys. Rev. C **72**, 035205 (2005).
 - [31] H. Mao, R. K. Su and W. Q. Zhao, Phys. Rev. C **74**, 055204 (2006).

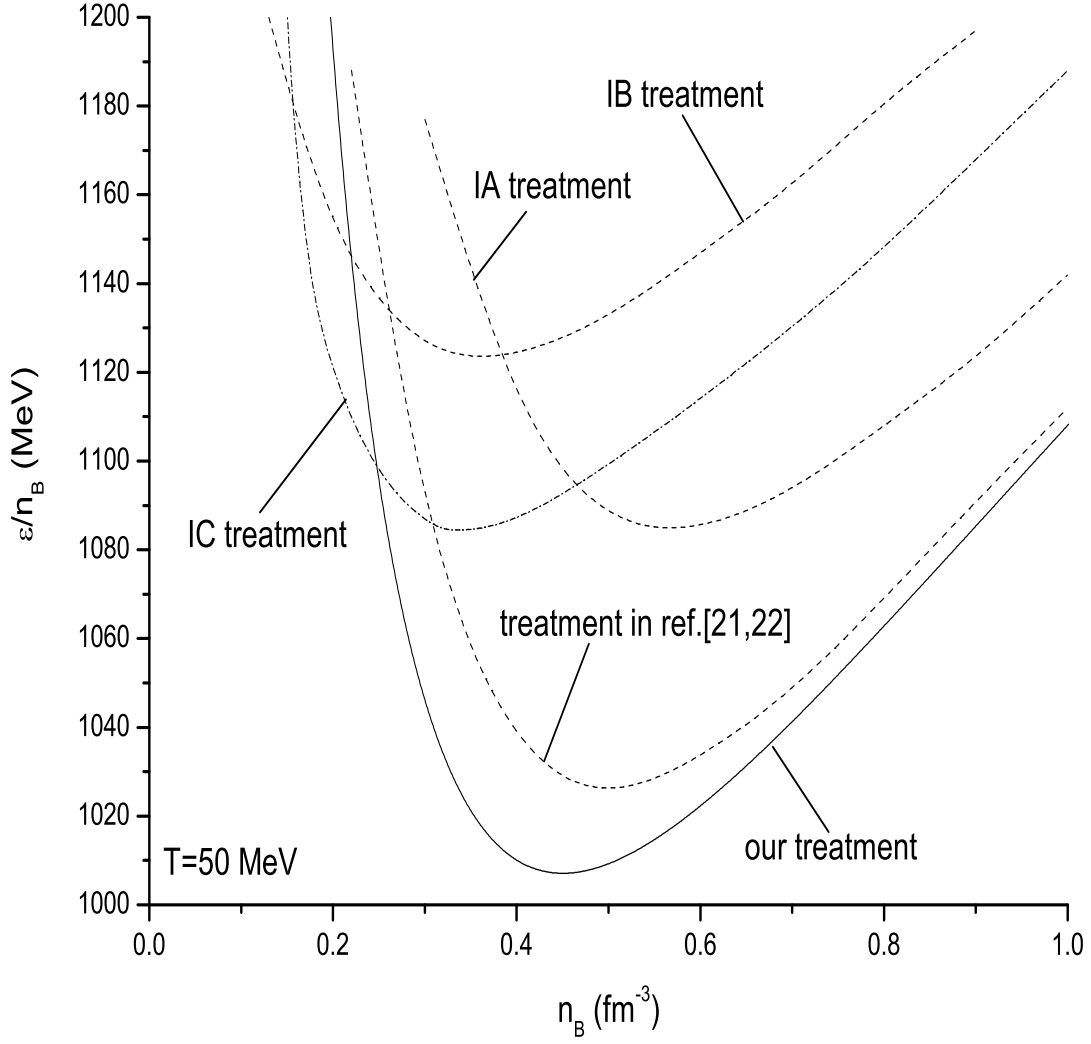


FIG. 1: Energy per baryon as a function of the baryon density n_B at $T = 50$ MeV for different treatments [28]. The treatment IC and our treatment are for the QMDTD model, and others are for the QMDD model.

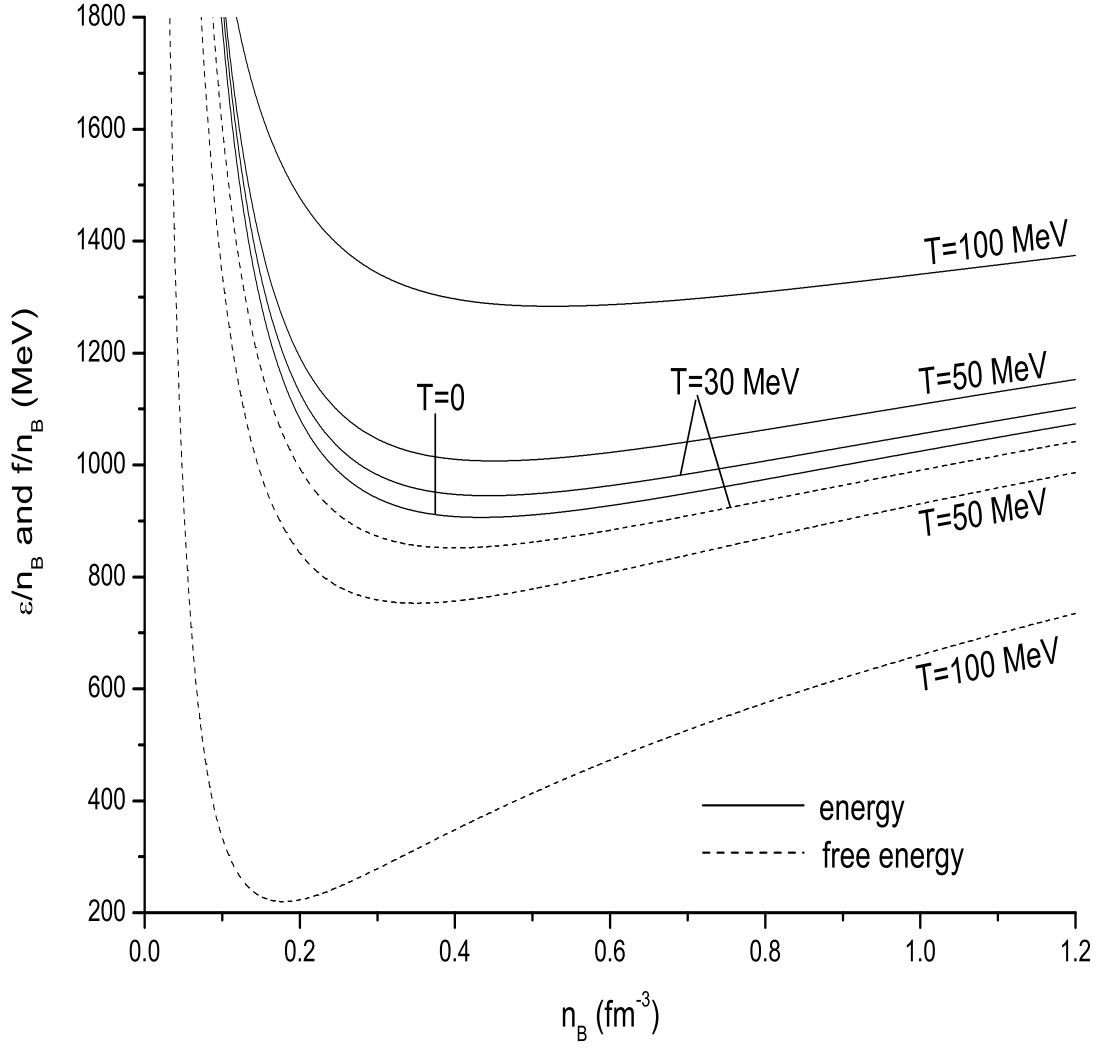


FIG. 2: Energy per baryon $U/N_B = \varepsilon/n_B$ and free energy per baryon $F/N_B = f/n_B$ as functions of the baryon density n_B at different temperatures $T = 0, 30, 50, 100$ MeV. The two curves are identical at $T = 0$.

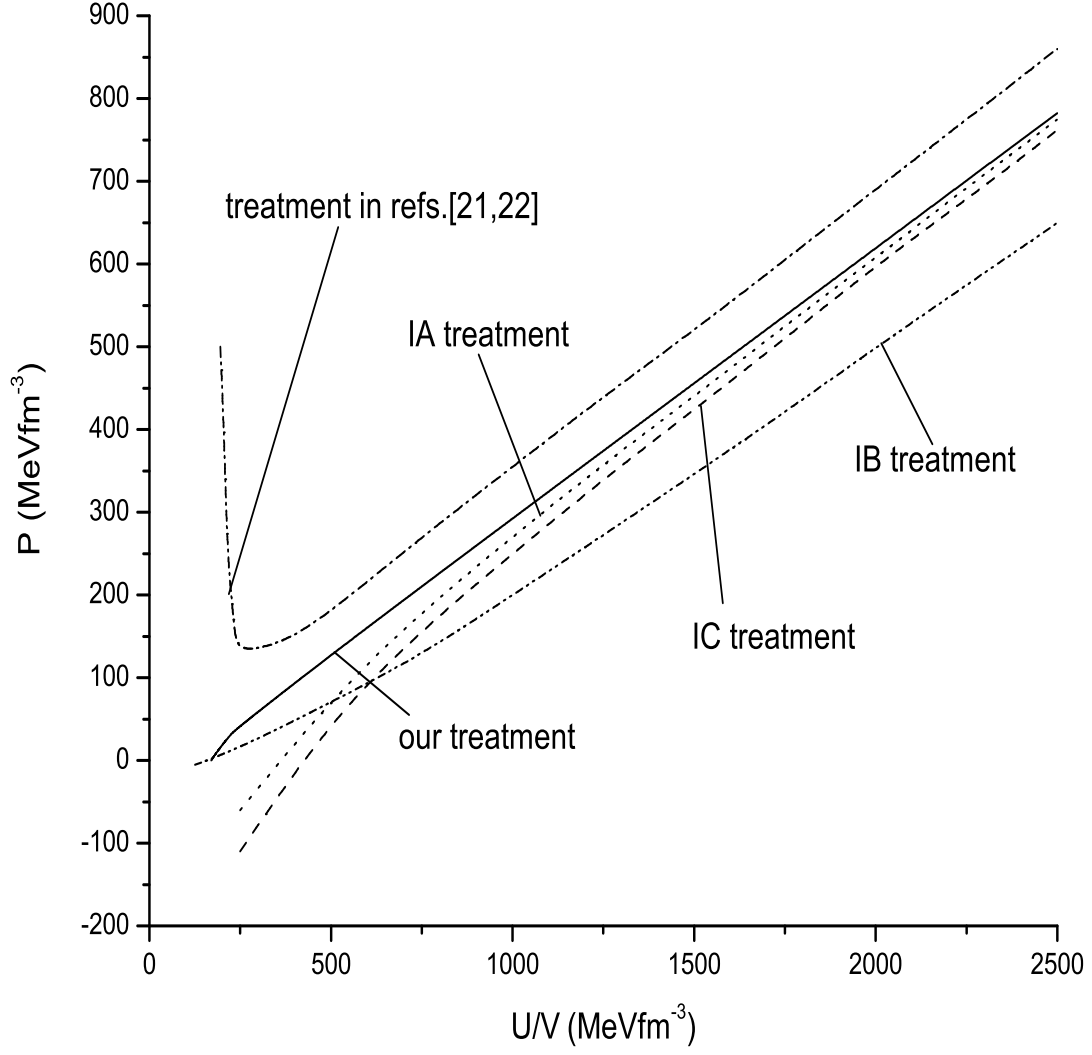


FIG. 3: Pressure P as a function of the energy density U/V for different treatments of both the QMDD and the QMDTD models [28]. The tendencies of the curves are similar at large energy density region, but at small energy density region, different treatments have quite different behaviors. In our treatment, the pressure never goes to negative.

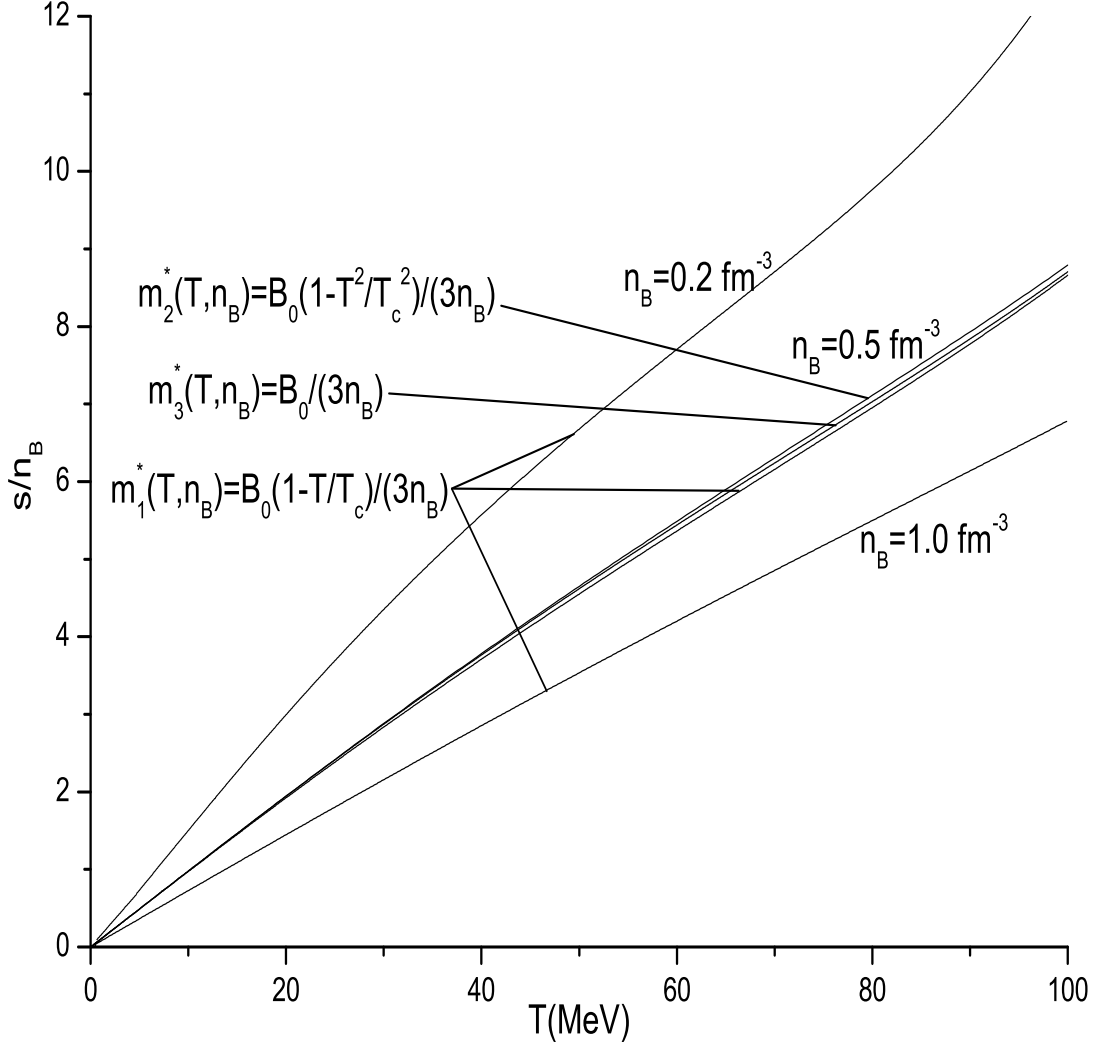


FIG. 4: Entropy per baryon vs. temperature curves for densities $n_B = 0.2, 0.5, 1.0 \text{ fm}^{-3}$, where the mass has been taken as $m_1^*(T, n_B)$, $m_2^*(T, n_B)$ and $m_3^*(T, n_B)$ respectively, as indicated in the figure. All curves get together at $S = 0$ when $T = 0$, which is consistent with the third law of thermodynamics.